

Testing Planarity by Switching Trains

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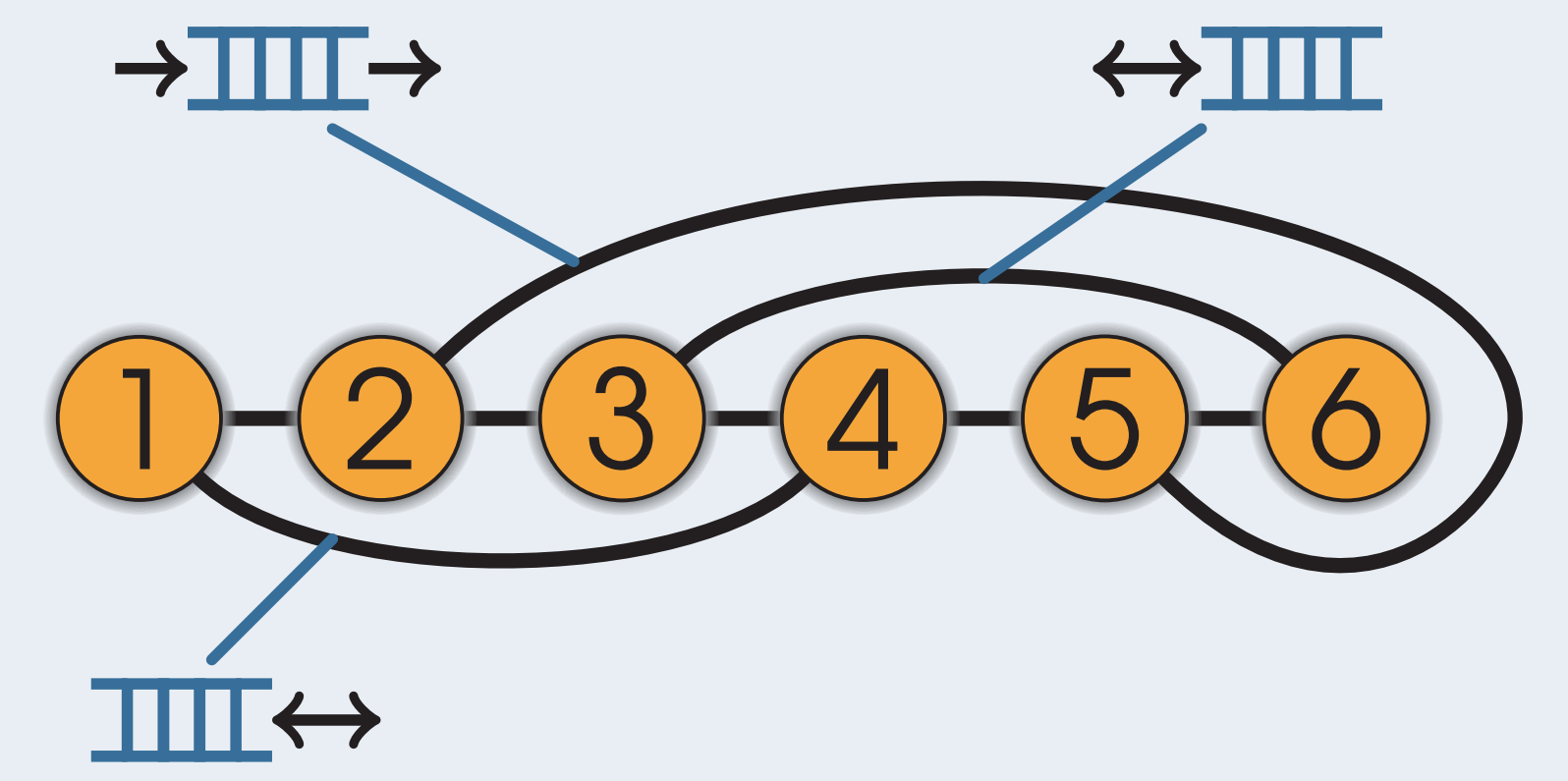
Results about Graph Layouts

A graph has a layout in... (cf. (1, 2))

- ▶ one stack \Leftrightarrow outerplanar
- ▶ two stacks \Leftrightarrow planar & Hamiltonian **circle**
- ▶ one deque \Leftrightarrow planar & Hamiltonian **path**

Theorem: one splittable deque \Leftrightarrow planar

Deque Layout



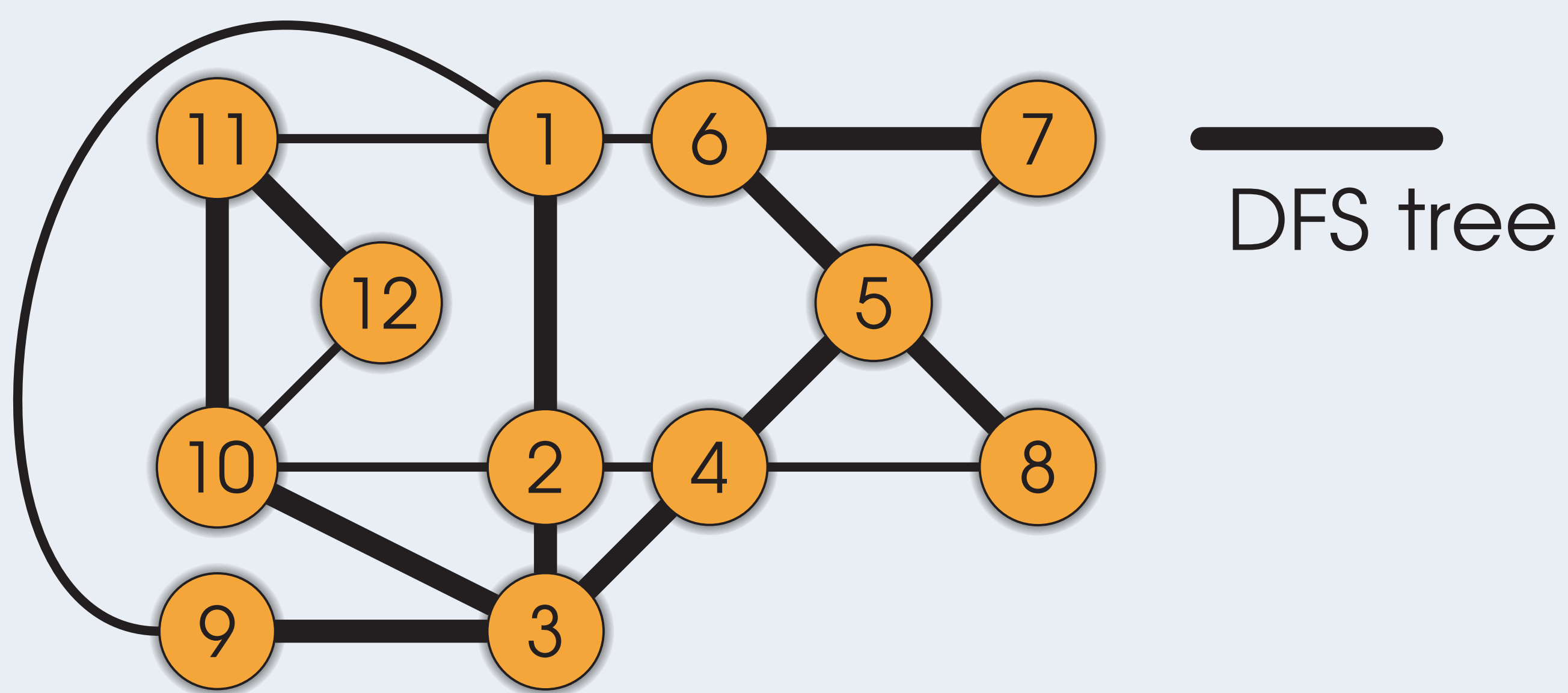
Splittable Deque

- ▶ DFS tree + DFS numbering v_1, \dots, v_n
- ▶ for each vertex $v_i = v_1, \dots, v_n$
 - ▶ **split** SD into k SDs (one per child)
 - ▶ **remove** edges from DFS ancestors of v_i from heads/tails of SDs
 - ▶ **prepend/append** edges to DFS descendants of v_i to SDs

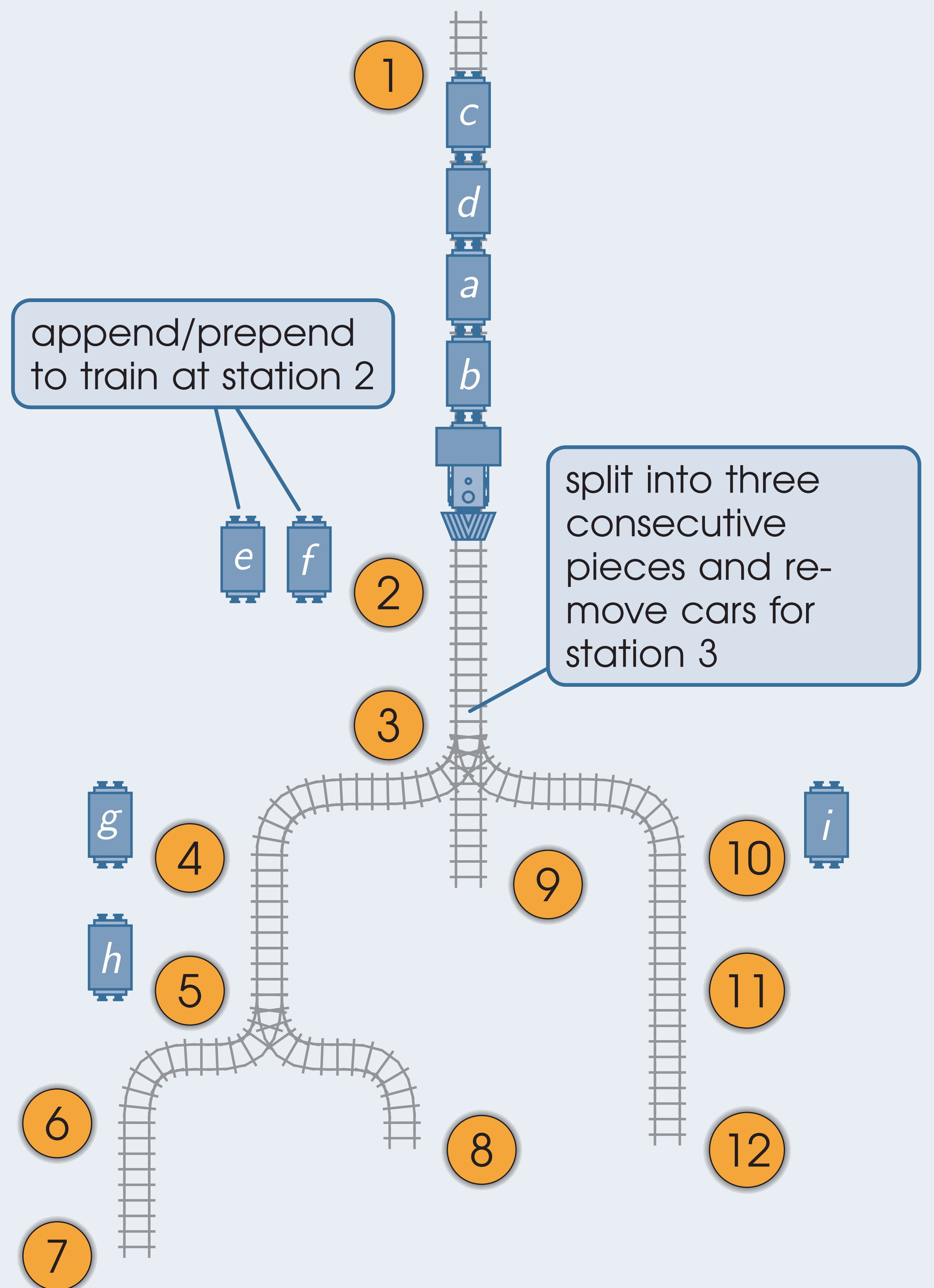
Train Switching Problem

- ▶ DFS tree \rightarrow rail network
- ▶ vertices \rightarrow stations; edges \rightarrow cars
- ▶ for each train station $s_i = s_1, \dots, s_n$
 - ▶ **split** train (one per junction)
 - ▶ **remove** cars with destination s_i
 - ▶ **prepend/append** cars with source s_i

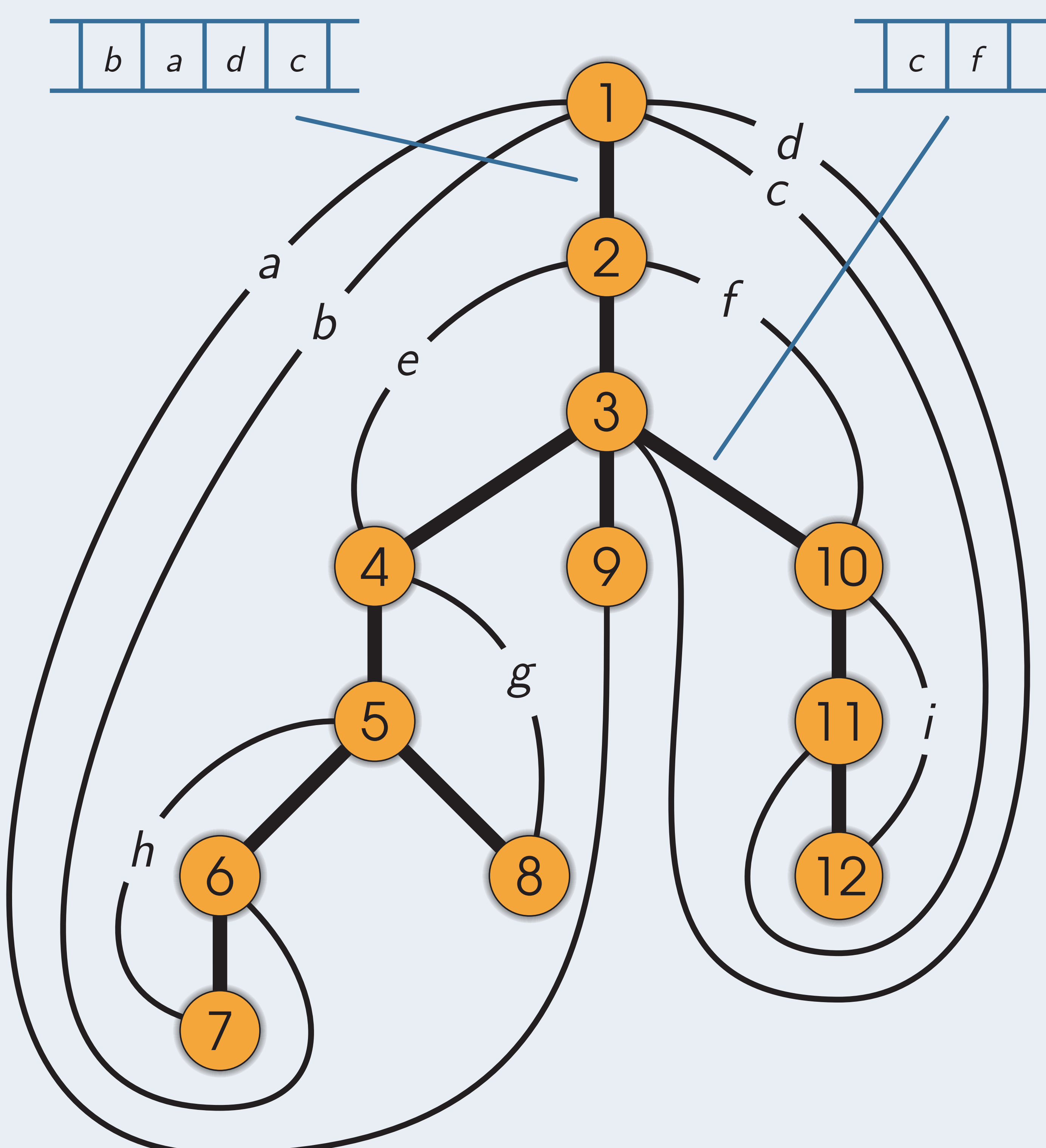
Planar Graph and DFS Tree



Train Switching Example



Splittable Deque Layout



References

- (1) Bernhart, F., Kainen, P.: The Book Thickness of a Graph. J. Combin. Theory, Ser. B27(3), 320–331 (1979)
- (2) Auer, C., Gleißner, A.: Characterizations of Queue and Deque Graphs. Proc. 37th International Conference on Graph-Theoretic Concepts in Computer Science (WG 2011), LNCS 6986, Springer Verlag, Heidelberg